Mid-Semestral Examination I Semester 2002-2003
B. Math. Hons. I Year
Algebra I

Date: 03-10-2002 **Max. Marks: 100** **Time: 3 Hours**

Note: Answer *all* questions. Yours answers should be clear and the arguments complete.

1. Define a group. Give an example of (a) an infinite family of abelian groups and (b) an infinite family of nonabelian groups. (4+6+6)

2. Let x and y be elements of a group of order m and n, respectively. Assume that $xy = yx$. Show that the order of xy ~~is~~ the least common divides multiple of m and n. Does S_{20} contain an element of order 56? (10+8)

3. (a) Show that the number of elements of a subgroup H of a finite group G is atmost half the number of elements of the group.

   (b) Show that H is normal in G if the order of H is half the order of the group G. (6+6)

4. (a) Define the conjugacy class of an element of a group and the partition of an integer $n \geq 0$. (4+4)

   (b) In S_n, prove *rigorously* that there is a bijection from the collection of the conjugacy classes of element of S_n and the partitions of n. (12)

   (c) In S_5, determine the number of conjugacy classes and the number of elements in each conjugacy class. (12)

5. Define an (i) automorphism of a group (ii) inner automorphism of a group. Calculate the inner automorphism group for each of the groups: $\mathbb{Z}_{10}$ and S_4. (4+4+2+4)

6. Let n be an integer. Show that if G contains a unique subgroup H of order n, then H is normal in G. (8)

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